

(20 points total)

Instructions:

- 1) The test is due on Wednesday February 18 before the start of class at 10:30 AM.
- 2) You must show all steps in your solutions
- 3) Your exam solutions are to be your own work, you are not to give or receive assistance of any kind on this exam.

1. (20 pts) Given the system

$$m\ddot{x} + b\dot{x}|\dot{x}| + k_0x + k_1x^3 = 0$$

where  $m, b, k_0, k_1 > 0$ .

- a) Find all equilibrium points
- b) Put in state-space form using states labeled  $x_1$  and  $x_2$ .
- c) Find  $\dot{V}$  for the Lyapunov function candidate

$$V = \frac{m}{2}x_2^2 + \frac{1}{2}k_0x_1^2 + \frac{1}{4}k_1x_1^4$$

- d) Use Invariant Set Theorem with  $V, \dot{V}$  to discuss the stability at  $(0,0)$ .
- e) Use Lasalle's Theorem with  $V, \dot{V}$  to discuss the stability at  $(0,0)$ .
- f) Let  $m=k_0=k_1=1$  and  $b=2$  to plot the phase-plane portrait for  $-3.5 < x_1 < 3.5$  and  $-3.5 < x_2 < 3.5$ . Show all equilibrium points in this range and plot enough trajectories (forward direction only in solver options) to show what is happening.